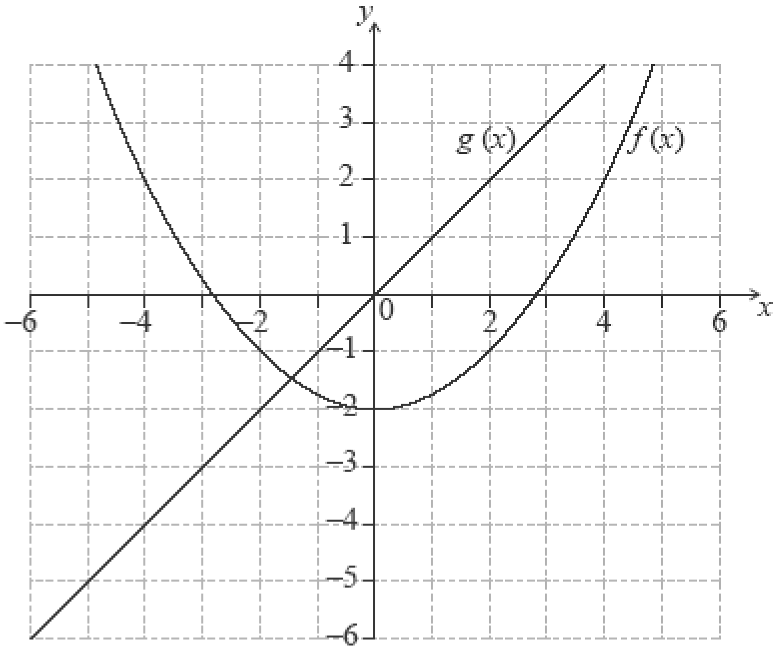
Calculus questions

1. The figure shows the graphs of the functions $f(x) = \frac{1}{4}x^2 - 2$ and $g(x) = x$.



(a) Differentiate $f(x)$ with respect to x.

(1)

(b) Differentiate $g(x)$ with respect to x.

(1)

(c) Calculate the value of x for which the gradients of the two graphs are the same.

(2)

(d) Draw the tangent to the parabola at the point with the value of x found in part (c).

(2)
(Total 6 marks)

2. Consider the function $f(x) = x^3 + \frac{48}{x}$, $x \neq 0$.

(a) Calculate $f(2)$.

(2)

(b) Sketch the graph of the function $y = f(x)$ for $-5 \leq x \leq 5$ and $-200 \leq y \leq 200$.

(4)

(c) Find $f'(x)$.

(3)

(d) Find $f'(2)$.

(2)

(e) Write down the coordinates of the local maximum point on the graph of f.

(2)

(f) Find the range of f.

(3)

(g) Find the gradient of the tangent to the graph of f at $x = 1$.

(2)

There is a second point on the graph of f at which the tangent is parallel to the tangent at $x = 1$.

(h) Find the x-coordinate of this point.

(2)
(Total 20 marks)

3. The function $f(x)$ is defined by $f(x) = 1.5x + 4 + \frac{6}{x}, x \neq 0$.

(a) Write down the equation of the vertical asymptote.

(2)

(b) Find $f'(x)$.

(3)

(c) Find the gradient of the graph of the function at $x = -1$.

(2)

(d) Using your answer to part (c), decide whether the function $f(x)$ is increasing or decreasing at $x = -1$. Justify your answer.

(2)

(e) Sketch the graph of $f(x)$ for $-10 \leq x \leq 10$ and $-20 \leq y \leq 20$.

(4)

P_1 is the local maximum point and P_2 is the local minimum point on the graph of $f(x)$.

(f) Using your graphic display calculator, write down the coordinates of

(i) P_1;

(ii) P_2.

(4)

(g) Using your sketch from (e), determine the range of the function $f(x)$ for $-10 \leq x \leq 10$.

(3)
(Total 20 marks)

4. The table given below describes the behaviour of $f'(x)$, the derivative function of $f(x)$, in the domain $-4 < x < 2$.

x	$f'(x)$
$-4 < x < -2$	< 0
-2	0
$-2 < x < 1$	> 0
1	0
$1 < x < 2$	> 0

(a) State whether $f(0)$ is greater than, less than or equal to $f(-2)$. Give a reason for your answer.

(2)

The point P(–2, 3) lies on the graph of $f(x)$.

(b) Write down the equation of the tangent to the graph of $f(x)$ at the point P.

(2)

(c) From the information given about $f'(x)$, state whether the point (–2, 3) is a maximum, a minimum or neither. Give a reason for your answer.

(2)

(Total 6 marks)

5. The diagram shows a sketch of the function $f(x) = 4x^3 - 9x^2 - 12x + 3$.

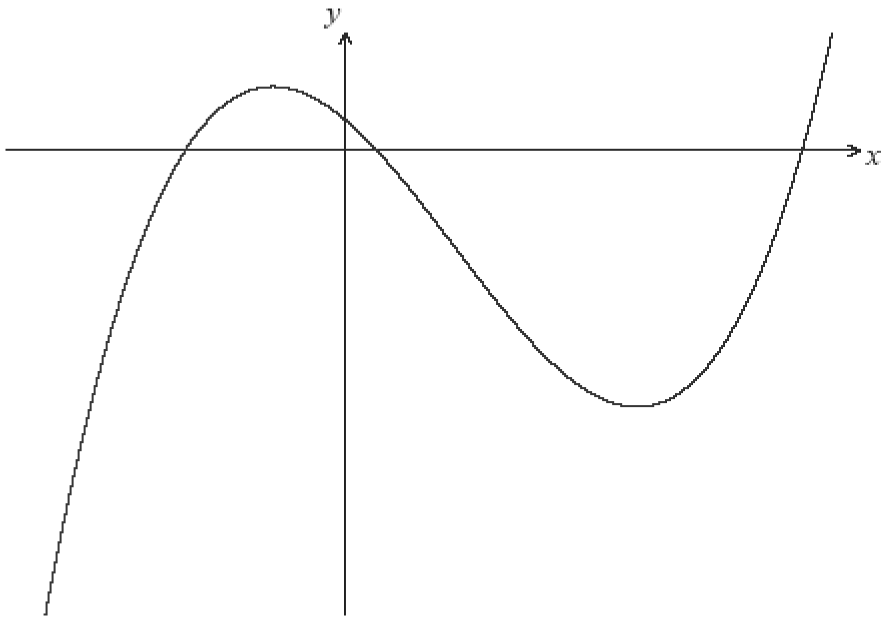


diagram not to scale

(a) Write down the values of x where the graph of $f(x)$ intersects the x-axis.

(3)

(b) Write down $f'(x)$.

(3)

(c) Find the value of the local maximum of $y = f(x)$.

(4)

Let P be the point where the graph of $f(x)$ intersects the y-axis.

(d) Write down the coordinates of P.

(1)

(e) Find the gradient of the curve at P.

(2)

The line, L, is the tangent to the graph of $f(x)$ at P.

(f) Find the equation of L in the form $y = mx + c$.

(2)

There is a second point, Q, on the curve at which the tangent to $f(x)$ is parallel to L.

(g) Write down the gradient of the tangent at Q.

(1)

(h) Calculate the x-coordinate of Q.

(3)
(Total 19 marks)

6. A dog food manufacturer has to cut production costs. She wishes to use as little aluminium as possible in the construction of cylindrical cans. In the following diagram, h represents the height of the can in cm, and x represents the radius of the base of the can in cm.

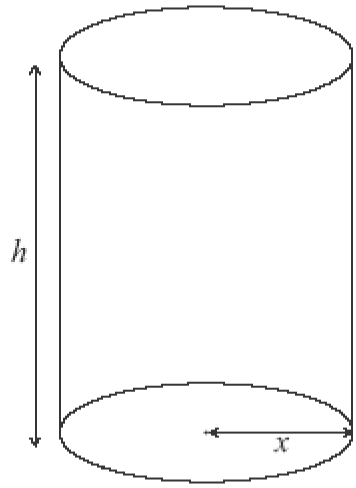


diagram not to scale

The volume of the dog food cans is 600 cm^3.

(a) Show that $h = \dfrac{600}{\pi x^2}$.

(2)

(b) (i) Find an expression for the curved surface area of the can, in terms of x. Simplify your answer.

(ii) Hence write down an expression for A, the total surface area of the can, in terms of x.

(4)

(c) Differentiate A in terms of x.

(3)

(d) Find the value of x that makes A a minimum.

(3)

(e) Calculate the minimum total surface area of the dog food can.

(2)
(Total 14 marks)

7. Consider the function $f(x) = x^3 - 3x^2 - 24x + 30$.

(a) Write down $f(0)$.

(1)

(b) Find $f'(x)$.

(3)

(c) Find the gradient of the graph of $f(x)$ at the point where $x = 1$.

(2)

The graph of $f(x)$ has a local maximum point, M, and a local minimum point, N.

(d) (i) Use $f'(x)$ to find the x-coordinate of M and of N.

(ii) Hence or otherwise write down the coordinates of M and of N.

(5)

(e) Sketch the graph of $f(x)$ for $-5 \le x \le 7$ and $-60 \le y \le 60$. Mark clearly M and N on your graph.

(4)

Lines L_1 and L_2 are parallel, and they are tangents to the graph of $f(x)$ at points A and B respectively. L_1 has equation $y = 21x + 111$.

(f) (i) Find the x-coordinate of A and of B.

(ii) Find the y-coordinate of B.

(6)
(Total 21 marks)

8. The straight line, L, has equation $2y - 27x - 9 = 0$.

(a) Find the gradient of L.

(2)

Sarah wishes to draw the tangent to $f(x) = x^4$ parallel to L.

(b) Write down $f'(x)$.

(1)

(c) (i) Find the x-coordinate of the point at which the tangent must be drawn.

(ii) Write down the value of $f(x)$ at this point.

(3)
(Total 6 marks)

9. Consider the function $f(x) = 3x + \dfrac{12}{x^2}$, $x \neq 0$.

(a) Differentiate $f(x)$ with respect to x.

(3)

(b) Calculate $f'(x)$ when $x = 1$.

(2)

(c) Use your answer to part (b) to decide whether the function, f, is increasing or decreasing at $x = 1$. Justify your answer.

(2)

(d) Solve the equation $f'(x) = 0$.

(3)

(e) The graph of f has a local minimum at point P. Let T be the tangent to the graph of f at P.

(i) Write down the coordinates of P.

(ii) Write down the gradient of T.

(iii) Write down the equation of T.

(5)

(f) Sketch the graph of the function f, for $-3 \le x \le 6$ and $-7 \le y \le 15$. Indicate clearly the point P and any intercepts of the curve with the axes.

(4)

(g) (i) On your graph draw and label the tangent T.

(ii) T intersects the graph of f at a second point. Write down the x-coordinate of this point of intersection.

(3)
(Total 22 marks)

10. Let $f(x) = 2x^2 + x - 6$

(a) Find $f'(x)$.

(3)

(b) Find the value of $f'(-3)$.

(1)

(c) Find the value of x for which $f'(x) = 0$.

(Total 6 marks)

11. The curve $y = px^2 + qx - 4$ passes through the point $(2, -10)$.

(a) Use the above information to write down an equation in p and q.

(2)

The gradient of the curve $y = px^2 + qx - 4$ at the point $(2, -10)$ is 1.

(b) (i) Find $\frac{dy}{dx}$.

(ii) Hence, find a second equation in p and q.

(3)

(c) Solve the equations to find the value of p and of q.

(3)
(Total 8 marks)

1. (a) $\frac{1}{2}x\left(\frac{2}{4}x\right)$ (A1) (C1)

Note: *Accept an equivalent, unsimplified expression (i.e. $2 \times \frac{1}{4}x$).*

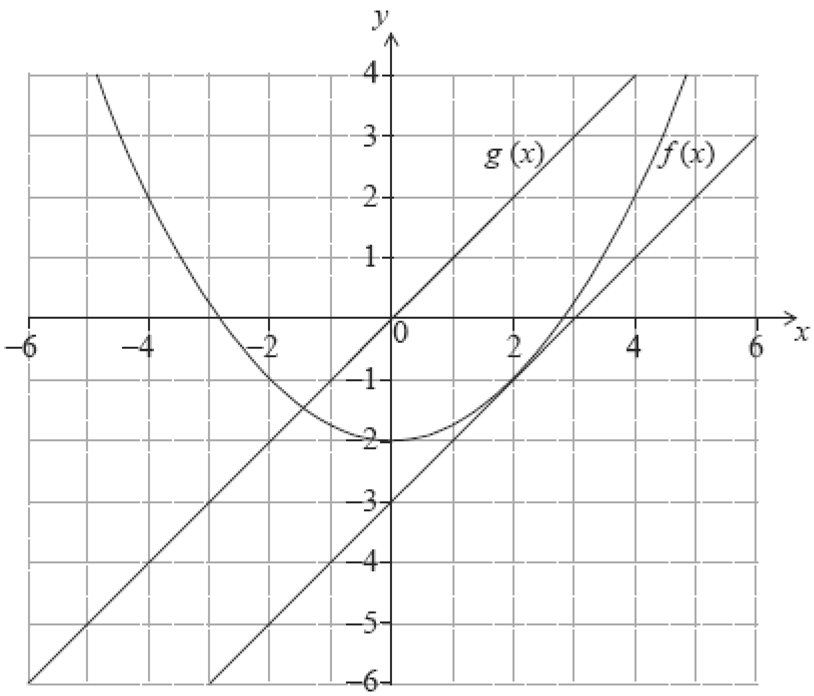
(b) 1 (A1) (C1)

(c) $\frac{1}{2}x = 1$ (M1)

$x = 2$ (A1)(ft) (C2)

Notes: *Award (M1)(A0) for coordinate pair (2, –1) seen with or without working.*
Follow through from their answers to parts (a) and (b).

(d)



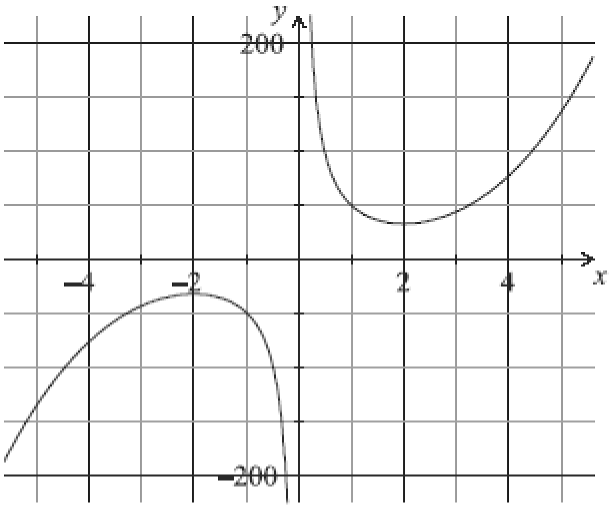
tangent drawn to the parabola at the x-coordinate found in part (c) (A1)(ft)
candidate's attempted tangent drawn parallel to the graph of $g(x)$ (A1)(ft) (C2)

[6]

2. (a) $f(2) = 2^3 + \frac{48}{2}$ (M1)

$= 32$ (A1)(G2)

(b)



(A1) for labels and some indication of scale in an appropriate window
(A1) for correct shape of the two unconnected and smooth branches
(A1) for maximum and minimum in approximately correct positions
(A1) for asymptotic behaviour at y-axis (A4)

***Notes:** Please be rigorous.*
The axes need not be drawn with a ruler.
The branches must be smooth: a single continuous line that does not deviate from its proper direction.
The position of the maximum and minimum points must be symmetrical about the origin.
*The y-axis must be an asymptote for **both** branches. Neither branch should touch the axis nor must the curve approach the asymptote then deviate away later.*

(c) $f'(x) = 3x^2 - \frac{48}{x^2}$ (A1)(A1)(A1)

***Notes:** Award (A1) for $3x^2$, (A1) for –48, (A1) for x^{-2}.*
Award a maximum of (A1)(A1)(A0) if extra terms seen.

(d) $f(2) = 3(2)^2 - \frac{48}{(2)^2}$ (M1)

***Note:** Award (M1) for substitution of x = 2 into their derivative.*

$= 0$ (A1)(ft)(G1)

(e) (–2,–32) or $x = -2$, $y = -32$ (G1)(G1)

***Notes:** Award (G0)(G0) for x = –32, y = –2*
Award at most (G0)(G1) if parentheses are omitted.

(f) $\{y > 32\} \cup \{y < -32\}$ (A1)(A1)(ft)(A1)(ft)

Notes: *Award (A1)(ft) $y \geq 32$ or $y > 32$ seen, (A1)(ft) for $y \leq -32$ or $y < -32$, (A1) for weak (non-strict) inequalities used in both of the above.*
Accept use of f in place of y. Accept alternative interval notation.
Follow through from their (a) and (e).
If domain is given award (A0)(A0)(A0).
Award (A0)(A1)(ft)(A1)(ft) for [–200, –32], [32, 200].
Award (A0)(A1)(ft)(A1)(ft) for]–200, –32], [32, 200[.

(g) $f'(1) = -45$ (M1)(A1)(ft)(G2)

Notes: *Award (M1) for f(1) seen or substitution of x = 1 into their derivative.*
Follow through from their derivative if working is seen.

(h) $x = -1$ (M1)(A1)(ft)(G2)

Notes: *Award (M1) for equating their derivative to their –45 or for seeing parallel lines on their graph in the approximately correct position.*

[20]

3. (a) $x = 0$ (A1)(A1)

Note: *Award (A1) for x = constant, (A1) for 0.*

(b) $f'(x) = 1.5 - \dfrac{6}{x^2}$ (A1)(A1)(A1)

Notes: *Award (A1) for 1.5, (A1) for –6, (A1) for x^{-2}*
Award (A1)(A1)(A0) at most if any other term present.

(c) $1.5 - \dfrac{6}{(-1)^2}$ (M1)

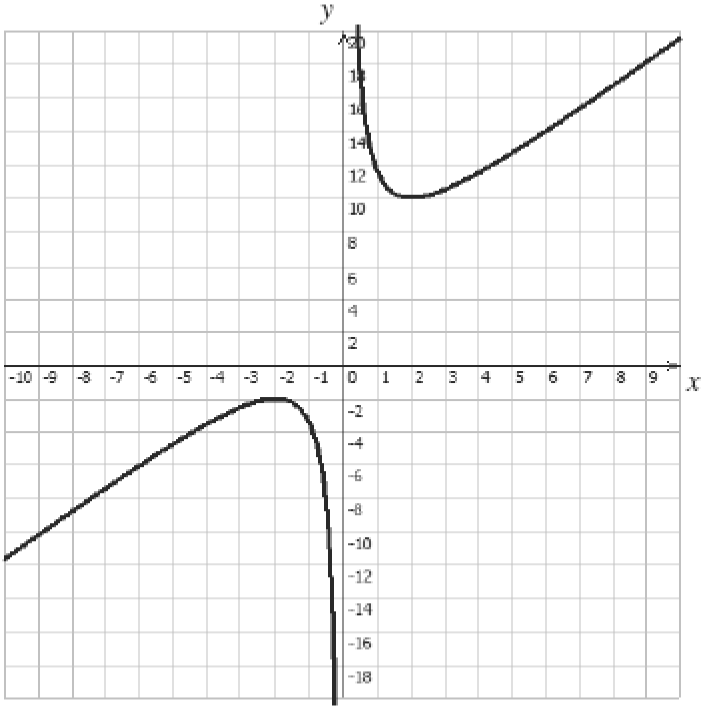
$= -4.5$ (A1)(ft)(G2)

Note: *Follow through from their derivative function.*

(d) Decreasing, the derivative (gradient or slope) is negative (at $x = -1$)(A1)(R1)(ft)

Notes: *Do not award (A1)(R0).*
Follow through from their answer to part (c).

(e)



(A4)

Notes: *Award (A1) for labels and some indication of scales and an appropriate window. Award (A1) for correct shape of the two unconnected, and smooth branches.*
Award (A1) for the maximum and minimum points in the approximately correct positions. Award (A1) for correct asymptotic behaviour at x = 0.

Notes: *Please be rigorous.*
The axes need not be drawn with a ruler.
The branches must be smooth and single continuous lines that do not deviate from their proper direction.
The max and min points must be symmetrical about point (0, 4).
*The y-axis must be an asymptote for **both** branches.*

(f) (i) (–2, – 2) or $x = -2$, $y = -2$ (G1)(G1)

(ii) (2, 10) or $x = 2$, $y = 10$ (G1)(G1)

(g) $\{-2 \geq y\}$ or $\{y \geq 10\}$ (A1)(A1)(ft)(A1)

Notes: *(A1)(ft) for $y > 10$ or $y \geq 10$*
(A1)(ft) for $y < -2$ or $y \leq -2$
*(A1) for weak (non-strict) inequalities used in **both** of the above.*
Follow through from their (e) and (f).

[20]

4. (a) greater than (A1)
Gradient between $x = –2$ and $x = 0$ is positive. (R1)

OR

The function is increased between these points or equivalent. (R1) (C2)

Note: *Accept a sketch. Do not award (A1)(R0).*

(b) $y = 3$ (A1)(A1) (C2)

Note: *Award (A1) for y = a constant, (A1) for 3.*

(c) minimum (A1)
Gradient is negative to the left and positive to the right or equivalent. (R1) (C2)

Note: *Accept a sketch. Do not award (A1)(R0).*

[6]

5. (a) −1.10, 0.218, 3.13 (A1)(A1)(A1)

(b) $f'(x) = 12x^2 – 18x – 12$ (A1)(A1)(A1)

Note: *Award (A1) for each correct term and award maximum of (A1)(A1) if other terms seen.*

(c) $f'(x) = 0$ (M1)
$x = –0.5, 2$
$x = –0.5$ (A1)

Note: *If x = –0.5 not stated, can be inferred from working below.*

$y = 4(–0.5)^3 – 9(–0.5)^2 – 12(–0.5) + 3$ (M1)
$y = 6.25$ (A1)(G3)

Note: *Award (M1) for their value of x substituted into f(x). Award (M1)(G2) if sketch shown as method. If coordinate pair given then award (M1)(A1)(M1)(A0)**.** If coordinate pair given with no working award (G2).*

(d) (0, 3) (A1)

Note: *Accept x = 0, y = 3.*

(e) $f'(0) = –12$ (M1)(A1)(ft)(G2)

Note: *Award (M1) for substituting x = 0 into their derivative.*

(f) Tangent: $y = -12x + 3$ (A1)(ft)(A1)(G2)

Note: *Award (A1)(ft) for their gradient, (A1) for intercept = 3. Award (A1)(A0) if y = not seen.*

(g) -12 (A1)(ft)

Note: *Follow through from their part (e).*

(h) $12x^2 - 18x - 12 = -12$ (M1)

$12x^2 - 18x = 0$ (M1)

$x = 1.5, 0$

At Q, $x = 1.5$ (A1)(ft)(G2)

Note: *Award (M1)(G2) for $12x^2 - 18x - 12 = -12$ followed by $x = 1.5$.*
Follow through from their part (g).

[19]

6. (a) $600 = \pi x^2 h$ (M1)(A1)

$\frac{600}{\pi x^2} = h$ (AG)

Note: *Award (M1) for correct substituted formula, (A1) for correct substitution. If answer given not shown award at most (M1)(A0).*

(b) (i) $C = 2\pi x \frac{600}{\pi x^2}$ (M1)

$C = \frac{1200}{x}$ (or $1200x^{-1}$) (A1)

Note: *Award (M1) for correct substitution in formula, (A1) for correct simplification.*

(ii) $A = 2\pi x^2 + 1200x^{-1}$ (A1)(A1)(ft)

Note: *Award (A1) for multiplying the area of the base by two, (A1) for adding on their answer to part (b)(i).*
For both marks to be awarded answer must be in terms of x.

(c) $\frac{dA}{dx} = 4\pi x - \frac{1200}{x^2}$ (A1)(ft)(A1)(ft)(A1)(ft)

Notes: *Award (A1) for $4\pi x$, (A1) for -1200, (A1) for x^{-2}. Award at most (A2) if any extra term is written. Follow through from their part (b)(ii).*

(d) $4\pi x - \frac{1200}{x^2} = 0$ (M1)(M1)

$x^3 = \dfrac{1200}{4\pi}$ (or equivalent)

$x = 4.57$ (A1)(ft)(G2)

Note: *Award (M1) for using their derivative, (M1) for setting the derivative to zero, (A1)(ft) for answer.*
Follow through from their derivative.
Last mark is lost if value of x is zero or negative.

(e) $A = 2\pi(4.57)^2 + 1200(4.57)^{-1}$ (M1)

$A = 394$ (A1)(ft)(G2)

Note: *Follow through from their answers to parts (b) (ii) and (d).*

[14]

7. (a) 30 (A1)

(b) $f'(x) = 3x^2 - 6x - 24$ (A1)(A1)(A1)

Note: *Award (A1) for each term. Award at most (A1)(A1) if extra terms present.*

(c) $f'(1) = -27$ (M1)(A1)(ft)(G2)

Note: *Award (M1) for substituting $x = 1$ into their derivative.*

(d) (i) $f'(x) = 0$

$3x^2 - 6x - 24 = 0$ (M1)

$x = 4$; $x = -2$ (A1)(ft)(A1)(ft)

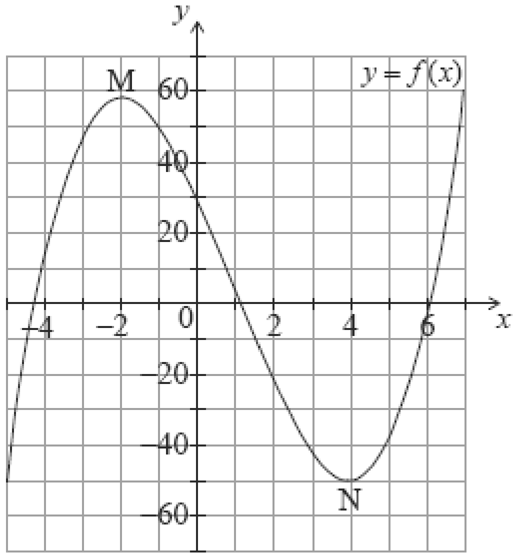
Notes: *Award (M1) for either $f'(x) = 0$ or $3x^2 - 6x - 24 = 0$ seen.*
Follow through from their derivative.
Do not award the two answer marks if derivative not used.

(ii) M(–2, 58) accept $x = -2$, $y = 58$ (A1)(ft)

N(4, –50) accept $x = 4$, $y = -50$ (A1)(ft)

Note: *Follow through from their answer to part (d) (i).*

(e)



(A1) for window
(A1) for a smooth curve with the correct shape
(A1) for axes intercepts in approximately the correct positions
(A1) for M and N marked on diagram and in approximately correct position (A4)

***Note:** If window is not indicated award at most* (A*0*)*(A1)*(A*0*)*(A1)(ft)*.

(f) (i) $3x^2 - 6x - 24 = 21$ (M1)

$3x^2 - 6x - 45 = 0$ (M1)

$x = 5$; $x = -3$ (A1)(ft)(A1)(ft)(G3)

***Note:** Follow through from their derivative.*

OR

Award (A1) for L_1 drawn tangent to the graph of f on their sketch in approximately the correct position ($x = -3$), (A1)(ft)

(A1) for a second tangent parallel to their L_1, (A1)(ft)

(A1) for $x = -3$, (A1) for $x = 5$. (A1)(A1)

***Note:** If only x = –3 is shown without working award (G2).*
If both answers are shown irrespective of working award (G3).

(ii) $f(5) = -40$ (M1)(A1)(ft)(G2)

***Notes:** Award (M1) for attempting to find the image of their x = 5. Award (A1) only for (5, – 40).*
Follow through from their x-coordinate of B ***only if it has been clearly identified*** *in (f) (i).*

[21]

8. (a) $y = 13.5x + 4.5$ (M1)

__Note:__ Award (M1) for 13.5x seen.

gradient = 13.5 (A1) (C2)

(b) $4x^3$ (A1) (C1)

(c) (i) $4x^3 = 13.5$ (M1)

__Note:__ Award (M1) for equating their answers to (a) and (b).

$x = 1.5$ (A1)(ft)

(ii) $\frac{81}{16}$ (5.0625, 5.06) (A1)(ft) (C3)

__Note:__ Award (A1)(ft) for substitution of their (c)(i) into x^4 with working seen.

[6]

9. (a) $f'(x) = 3 - \frac{24}{x^3}$ (A1)(A1)(A1)

__Note:__ Award (A1) for 3, (A1) for –24, (A1) for x^3 (or x^{-3}). If extra terms present award at most (A1)(A1)(A0).

(b) $f'(1) = -21$ (M1)(A1)(ft)(G2)

__Note:__ (ft) from their derivative only if working seen.

(c) Derivative (gradient, slope) is negative. Decreasing. (R1)(A1)(ft)

__Note:__ Do not award (R0)(A1).

(d) $3 - \frac{24}{x^3} = 0$ (M1)

$x^3 = 8$ (A1)

$x = 2$ (A1)(ft)(G2)

(e) (i) (2, 9) (Accept $x = 2$, $y = 9$) (A1)(A1)(G2)

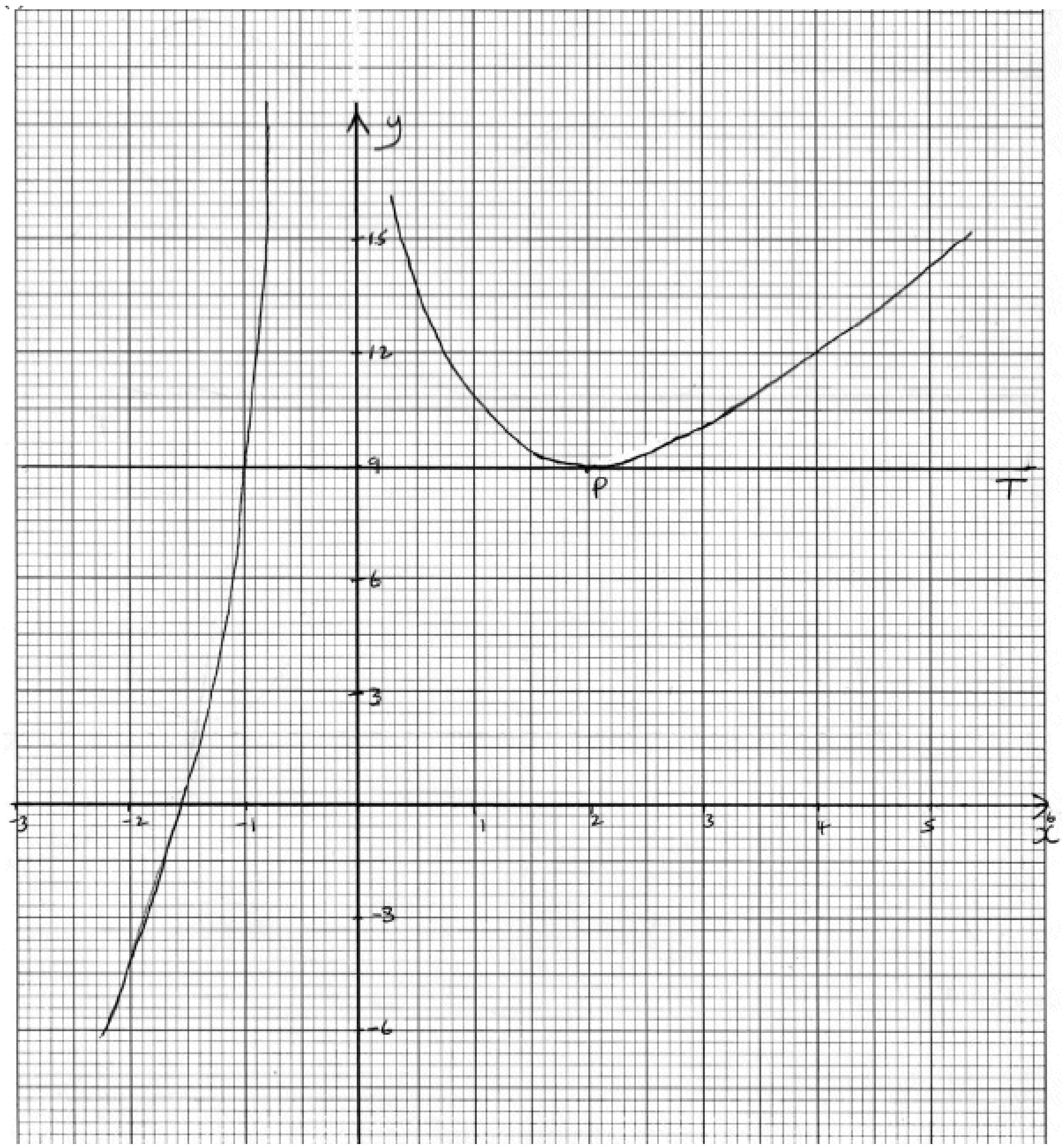
__Notes:__ (ft) from their answer in (d).
Award (A1)(A0) if brackets not included and not previously penalized.

(ii) 0 (A1)

(iii) $y = 9$ (A1)(A1)(ft)(G2)

Notes: *Award (A1) for y = constant, (A1) for 9.*
Award (A1)(ft) for their value of y in (e)(i).

(f)



(A4)

Notes: *Award (A1) for labels and some indication of scale in the stated window.*
Award (A1) for correct general shape (curve must be smooth and must not cross the y-axis).
Award (A1) for x-intercept seen in roughly the correct position.
Award (A1) for minimum (P).

(g) (i) Tangent drawn at P (line must be a tangent and horizontal). (A1)
Tangent labeled T. (A1)

(ii) $x = -1$ (G1)(ft)

Note: *(ft) from their tangent equation only if tangent is drawn and answer is consistent with graph.*

[22]

10. (a) $f'(x) = 4x + 1$ (A1)(A1)(A1) (C3)

Note: *Award (A1) for each term differentiated correctly. Award at most (A1)(A1)(A0) if any extra terms seen.*

(b) $f'(-3) = -11$ (A1)(ft) (C1)

(c) $4x + 1 = 0$ (M1)

$x = -\frac{1}{4}$ (A1)(ft) (C2)

[6]

11. (a) $2^2 \times p + 2q - 4 = -10$ (M1)

Note: *Award (M1) for correct substitution in the equation.*

$4p + 2q = -6$ or $2p + q = -3$ (A1)

Note: *Accept equivalent simplified forms.*

(b) (i) $\frac{dy}{dx} = 2px + q$ (A1)(A1)

Note: *Award (A1) for each correct term. Award at most (A1)(A0) if any extra terms seen.*

(ii) $4p + q = 1$ (A1)(ft)

(c) $4p + 2q = -6$
$4p + q = 1$ (M1)

Note: *Award (M1) for sensible attempt to solve the equations.*

$p = 2, q = -7$ (A1)(A1)(ft)(G3)

[8]

1. **Calculus and tangent drawing**

Parts (a) and (b) were reasonably well attempted indicating that candidates are well drilled in the process of differentiation. Correct answers however in part (c) proved elusive to many as frequent attempts to equate the two given functions rather than the gradients of the given functions resulted in a popular, but incorrect, answer of $x = -1.46$. Part (d) was poorly attempted with many candidates simply either not attempting to draw a tangent or drawing it in the wrong place.

2. **Curve sketching and Differential Calculus**

As usual and by intention, this question caused the most difficulty in terms of its content; however, for those with a sound grasp of the topic, there were many very successful attempts. Much of the question could have been answered successfully by using the GDC, however, it was also clear that a number of candidates did not connect the question they were attempting with the curve that they had either sketched or were viewing on their GDC. Where there was no alternative to using the calculus, many candidates struggled.

The majority of sketches were drawn sloppily and with little attention to detail. Teachers must impress on their students that a mathematical sketch is designed to illustrate the main points of a curve – the smooth nature by which it changes, any symmetries (reflectional or rotational), positions of turning points, intercepts with axes and the behaviour of a curve as it approaches an asymptote. There must also be some indication of the dimensions used for the "window".

Differentiation of terms with negative indices remains a testing process for the majority; it will continue to be tested.

It was also evident that some centres do not teach the differential calculus.

3. **Calculus**

Part a) was either answered well or poorly. Most candidates found the first term of the derivative in part b) correctly, but the rest of the terms were incorrect. The gradient in c) was for the most part correctly calculated, although some candidates substituted incorrectly in $f(x)$ instead of in $f'(x)$. Part d) had mixed responses. Lack of labels of the axes, appropriate scale, window, incorrect maximum and minimum and incorrect asymptotic behaviour were the main problems with the sketches in e). Part f) was also either answered correctly or entirely incorrectly. Some candidates used the trace function on the GDC instead of the min and max functions, and thus acquired coordinates with unacceptable accuracy. Some were unclear that a point of local maximum may be positioned on the coordinate system "below" the point of local minimum, and exchanged the pairs of coordinates of those points in f(i) and f(ii). Very few candidates were able to identify the range of the function in (g) irrespective of whether or not they had the sketches drawn correctly.

4. **Calculus**

Very few candidates received full marks for this question and many omitted the question completely. A sketch showing the information provided in the table would have been very useful but few candidates chose this approach.

5. **Calculus**

This question was either very well done – by the majority – or very poor and incomplete attempts were seen. This would perhaps indicate a lack of preparation in this area of the syllabus from some centres, though it is recognised that the differential calculus is one of the more problematic topics for the candidature.

It was however disappointing to note the number of candidates who do not use the GDC to good effect; in part (a) for example, the zeros were not found accurately due to "trace" being used; this is not a suitable approach – there is a built-in zero finder which should be used. Much of the question was accessible via a GDC approach, a sketch was given that could have been verified on the GDC; this was lost on many.

6. Optimization

This was the most difficult question for the candidates. It was clear that the vast majority of them had not had exposure to this style of question. Part (a) was well answered by most of the students. In part (b) the correct expression "in terms of x" for the curve surface area was not frequently seen. In many cases the impression was that they did not know what "in terms of x" meant as correct equivalent expressions were seen but where the h was also involved. Those candidates that made progress in the question, even with the wrong expression for the total area of the can, A were able to earn follow through marks.

7. Calculus

The value of $f(0)$ and the derivative function $f'(x)$, were well done in parts (a) and (b). In part (c) many candidates found $f(1)$ instead of $f'(1)$. In part (d) many students did not use their $f'(x)$ to find the x-coordinates of M and N and instead used their GDC. The sketch was generally well done although some students forgot to label M and N or did not use the specified window. The last part of the question was a clear discriminator. Examiners were pleased to see how this challenging question was solved using alternative methods.

8. Differentiation

The structure of this question was not well understood by the majority; the links between parts not being made. Again, this question was included to discriminate at the grade 6/7 level.

(a) Most were successful in this part.

(b) This part was usually well attempted.

(c) Only the best candidates succeeded in this part.

9. Calculus

Many students did not know the term "differentiate" and did not answer part (a). However, the derivative was seen in (b) when finding the gradient at $x = 1$. The negative index of the formula did cause problems for many when finding the derivative. The meaning of the derivative was not clear for a number of students. Part (d) was handled well by some but many substituted $x = 0$ into $f'(x)$. It was clear that most candidates neither knew that the tangent at a minimum is horizontal nor that its gradient is zero. There were good answers to the sketch though setting out axes and a scale seemed not to have had enough practise.

Those who were able to sketch the function were often able to correctly place and label the tangent and also to find the second intersection point with the graph of the function.

10. Calculus

This was a fairly standard question. However, some candidates found $f(–3)$ instead of $f'(–3)$. Quite a few candidates were unable to answer part (c) as they tried to find $f'(0)$ instead of finding x when $f'(x) = 0$.

11. Curve sketching and differential calculus

Undoubtedly, this question caused the most difficulty in terms of its content. Where there was no alternative to using the calculus, the majority of candidates struggled. However, for those with a sound grasp of the topic, there were many very successful attempts.

This question was challenging to the majority, with a large number not attempting the question at all. However, there were a pleasing number of correct attempts that showed a fine understanding of the calculus.